

## Translations of Linear & Exponential Functions

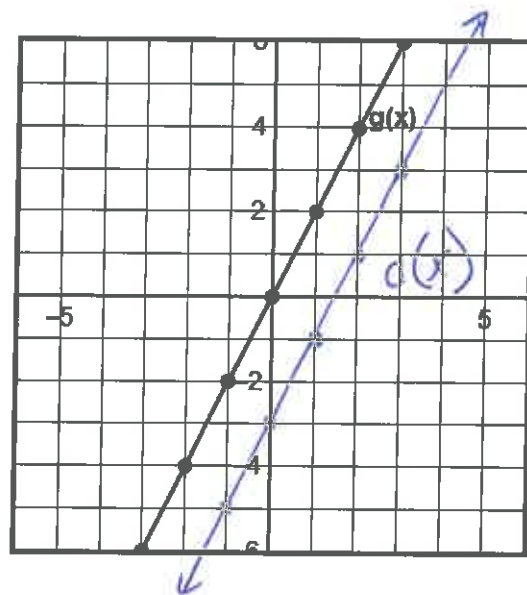
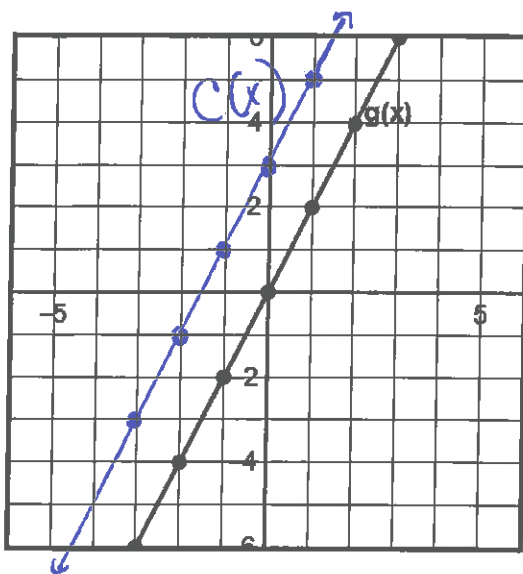


### PROBLEM 1 Vertical Translations

**Transformation:** The mapping, or movement, of all points of a figure in a plane according to a common operation.

**Vertical Translation:** A type of *transformation* that shifts an entire graph up or down. It affects only the y-coordinate of each point on the graph.

1. Consider the graph of a function  $g(x)$  on the two grids below.



- a. On the left grid, graph the function,  $c(x)$ , where  $c(x) = g(x) + 3$ . Describe how each point on  $g(x)$  changed to produce the graph of  $c(x)$ .

Added 3 to the y-coordinate, making the points move up.

- b. On the right grid, graph the function,  $d(x)$ , where  $d(x) = g(x) - 3$ . Describe how each point on  $g(x)$  changed to produce the graph of  $d(x)$ .

Subtracted 3 from the y-coordinate, making the points move down.

- c. If  $g(x) = 2x$ , then write the functions  $c(x)$  and  $d(x)$  in terms of  $x$ .

$$\begin{aligned} c(x) &= g(x) + 3 \\ &= 2x + 3 \end{aligned}$$

$$\begin{aligned} d(x) &= g(x) - 3 \\ &= 2x - 3 \end{aligned}$$

- d. Describe how the formula for  $g(x)$  must be changed to produce a vertical translation of its graph.

Add to  $g(x)$  to move it up.

Subtract from  $g(x)$  to move it down.

2. For each function, describe the transformation to the function  $s(x)$ .


 a.  $h(x) = s(x) + 8$   $s(x)$  shifted up 8 units.

b.  $j(x) = s(x) - 5$   $s(x)$  shifted down 5 units.

So, if a constant is added or subtracted OUTSIDE a function, like  $g(x) + 3$  or  $g(x) - 3$ , then only the  $y$ -values change, resulting in a vertical translation.



3. Write each function in terms of the function,  $f(x)$ , where  $f(x) = 2^x$ .

 a.  $h(x) = 2^x + 7$   
 $= f(x) + 7$

b.  $p(x) = 2^x - 4$   
 $= f(x) - 4$

c.  $n(x)$  is  $f(x)$  shifted down 5 units.

$n(x) = f(x) - 5$

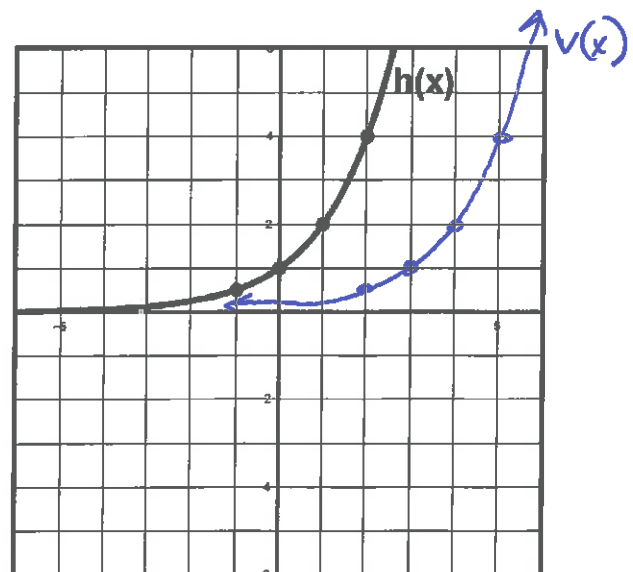
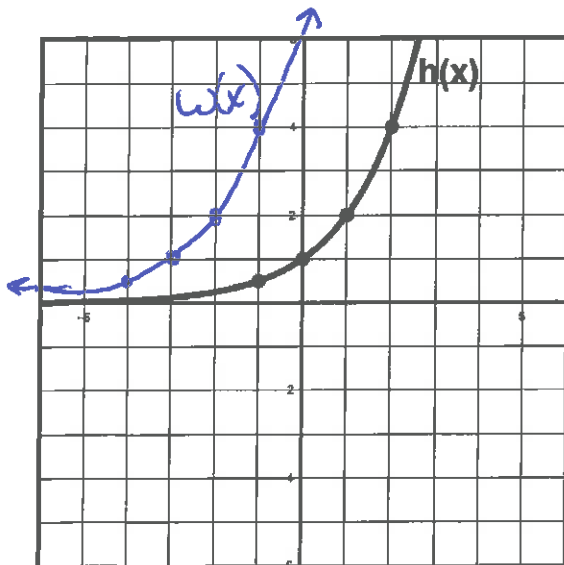
d.  $w(x)$  is  $f(x)$  shifted up 10 units.

$w(x) = f(x) + 10$

## PROBLEM 2 Horizontal Translations

**Horizontal Translation:** A type of transformation that shifts an entire graph left or right. It affects only the  $x$ -coordinate of each point on the graph.

1. Consider the graph of a function  $h(x)$  on the two grids below.



a. On the left grid, graph the function,  $w(x)$ , where  $w(x) = h(x+3)$ . Describe how each point on  $h(x)$  changed to produce the graph of  $w(x)$ .

Subtracted 3 from the  $x$ -coordinate making

each pt move left

b. On the right grid, graph the function,  $v(x)$ , where  $v(x) = h(x-3)$ . Describe how each point on  $h(x)$  changed to produce the graph of  $v(x)$ .

Add 3 to each  $x$ -coordinate making each pt move right.

c. If  $h(x) = 2^x$ , write the functions  $w(x)$  and  $v(x)$  in terms of  $x$ .

$$w(x) = h(x+3) \\ = 2^{(x+3)}$$

$$v(x) = h(x-3) \\ = 2^{(x-3)}$$

d. Describe how the formula for  $h(x)$  must be changed to produce a horizontal translation of its graph.

Subtract from  $x$  inside ( ) to move right

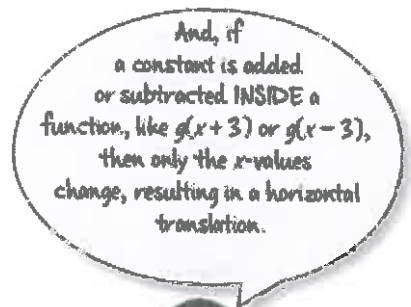
Add to  $x$  inside the ( ) to move left.

2. For each function, describe the transformation to the function  $s(x)$ .



a.  $h(x) = s(x+8)$   $s(x)$  shifted left 8 units

b.  $j(x) = s(x-5)$   $s(x)$  shifted Right 5 units.



3. Write each function in terms of the function,  $f(x)$ , where  $f(x) = 2^x$ .



a.  $h(x) = 2^{x+7}$

$$h(x) = f(x+7)$$

b.  $p(x) = 2^{x-4}$

$$p(x) = f(x-4)$$

c.  $n(x)$  is  $f(x)$  shifted right 5 units.

$$n(x) = f(x-5)$$

d.  $w(x)$  is  $f(x)$  shifted left 10 units.

$$w(x) = f(x+10)$$



## Check For Understanding:

Complete the table by describing the transformation of the graph of the basic function.

Function	Description of Transformation of Graph
$f(x) = 5x + 4$	the graph of $5x$ is shifted up 4 units.
$f(x) = 3(4^{x-6})$	the graph of $3(4^x)$ is shifted Right 6 units.
$f(x) = 8(x+2) - 9$	the graph of $8x$ is shifted down 9 and left 2 units.
$f(x) + a$ where $a > 0$	$f(x)$ is shifted up $a$ units
$f(x) - a$ where $a > 0$	$f(x)$ is shifted down $a$ units
$f(x+a)$ where $a > 0$	$f(x)$ is shifted left $a$ units
$f(x - a)$ where $a > 0$	$f(x)$ is shifted Right $a$ units.



Be prepared to share your answers.